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Performance Measurement of VA Hedging Strategies
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# Performance Measurement of VA Hedging Strategies

- **Insurers embed complex short equity put option positions in their Variable Annuity (VA) products.**
- **Expected hedging costs offset risk reduction benefits in the presence of equity market risk premia.**
- **This paper develops a consistent framework to compare all possible hedging strategies.**
- **Our analysis ranks hedging strategies and optimizes hedge parameter selection.**

## INTRODUCTION

The complex hybrid equity and interest rate options embedded in variable annuity (VA) products present formidable hedging challenges for the insurers who write them. Actuarial risks of policyholder behavior complicate this problem further. Few insurers have developed complete liability valuation models integrating all these factors. Yet, growth in the VA markets requires not only comprehensive valuation models, but also a means to measure the prospective performance of different hedging programs around these risks, and a way to help insurers decide how they are going to hedge.

**Equity risk premia dramatically affect hedge performance.**

This paper focuses on the performance measurement of VA hedging strategies and the selection of a hedging policy. Using a simplified liability model, we propose a technique to compare full insurance, static hedging, dynamic delta hedging and no insurance on an equal basis. The model considers the market risk premia for equity and volatility positions, but does not consider transaction costs explicitly. Also, the model presented here does not consider actuarial risks or interest rate risks, only equity risks. We also do not include capital, reserves or cash funding requirements of the different strategies. Nevertheless, the proposed technique can be used to incorporate these real-world factors in a straightforward manner.

This paper focuses on developing and explaining a tractable methodology for evaluating VA hedge programs but refrains from recommending a particular hedging strategy per se. As we explore one simple example, we must also be reminded that the results depend on the parameters chosen, particularly transaction costs.

## A Simple Example

**A simple example, but a robust methodology.**

In every policy, the VA writer sells an embedded equity put option to the insured party. The option is tremendously complicated, with lookback, minimum guarantee, ratchet and withdrawal features. Nevertheless, to illustrate our methodology we consider the case where the insurer has sold a single, simple equity put option. The basic methodology can accommodate more complicated put option structures.

**The insurer sells a single put option…**

In this case, the option strike and the underlying equity index value are taken to be 100. The option is assumed to expire in one year, and the corresponding risk-free rate is 6%. We assume the conditions of the Black-Scholes financial model in order to facilitate computation and exposition. The instantaneous growth of the index value is taken to be 15% per annum, with volatility of 18% per annum. This 18% volatility is termed the *actuarial volatility*, because it will be used for actuarial valuation of the liability.

**Retail value exceeds the actuarial value and the reinsurance value of the put.**

The presence of risk premia in long-dated equity puts has been well-documented. In this particular model, we capture the risk premium in volatility by setting an implied volatility for the option that exceeds the actuarial volatility. The implied volatility is assumed arbitrarily to be 21%. This can also be called the reinsurance volatility, since it reflects the price the insurer would pay to cede the risk to the marketplace. Finally, we presume the customer is paying a price for the option that exceeds implied volatility, though this need not necessarily be the case.[1] This *retail volatility* is taken to be 23%.

The embedded put options can be summarized in Table 1.

| | Volatility | Option value |
|---|---:|---:|
| Actuarial | 18% | $4.43 |
| Implied | 21% | 5.53 |
| Customer | 23% | 6.28 |

**Table 1: Embedded put options, Black-Scholes model**

Our analysis does not hinge on the Black-Scholes model, but does require the use of a pricing model that can value the liability and determine greeks[2] under different parameter assumptions.

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[1] The insurer can still make a profit if the retail volatility falls between the actuarial volatility and the reinsurance volatility, but it cannot lock in a positive profit.

[2] By "greeks" we mean the partial derivatives of the liability value with respect to underlying risk factors, including second derivatives and cross derivative terms. The delta is the first derivative of the liability value (in this case the embedded put) with respect to the underlying equity index. A "delta hedging strategy" implies a frequently rebalanced underlying equity position where the size of the position is taken to be the delta of the liability.

In simple terms, the insurance company in this example sells an option worth $4.43 to a customer for $6.28, creating a potential expected profit margin of $1.85, albeit with a great deal of risk. If the insurer *reinsures* by going to the option market, it can lock in a certain profit of $0.75.

The question is, how should the insurer hedge its sold option? Should it pay for expensive reinsurance? Should it try to create the actuarial option using delta hedging? Or should it forego hedging at all?

## ALTERNATIVES FOR HEDGING VA EMBEDDED OPTIONS

The options for hedging are wide-ranging. On one extreme, the insurer could decide not to hedge at all. On the other extreme, it can reinsure fully by purchasing equity puts in the marketplace. We consider examples of two other alternatives. One alternative is a static hedge, one that is placed in the futures or options market for the duration of the contract. Another alternative is the dynamic hedge, which requires frequent rebalancing as the delta of the underlying liability changes.

**Company reinsurance, static hedges and dynamic hedges.**

In practice, the static hedge will normally involve both traded options and futures. We consider a futures hedge for simplicity here in order to illustrate the method. In our example, the futures price can be calculated at inception according to the usual arbitrage-free futures pricing model:

$$\text{Futures price} = \text{Spot price} \times \exp(rt)$$

Assuming no dividends, we have a futures price equal to $106.18. We chose a static hedge ratio of -0.35 units of the underlying, in order to match the initial delta of the embedded put option.

In the dynamic hedging strategy, we propagate the futures price forward using the usual Black-Scholes architecture, and the actuarial growth and volatility:

$$S_{t+\Delta} = S_t \exp[(g - ½\sigma^2)\Delta + z\sigma\sqrt{\Delta}]$$

For each futures path, we compute the new option delta from Black-Scholes based on the new underlying value and time to maturity, and rebalance the delta hedge weekly. We compute the P&L from the change in the futures price along the path from week to week. Finally, the P&L is assumed to be invested at the risk-free rate so we thereby determine the proceeds at maturity.

In fact, all the strategies are evaluated carefully to determine the payout at maturity along each simulated path. All interim proceeds or costs of the hedging strategies are projected forward at the risk-free rate of interest. Even the full reinsurance strategy, though it locks in a positive net premium today, is evaluated based on investing the net option premium at the risk-free rate for the duration of the contract.

## THE EQUITY RISK PREMIA

We are comparing strategy performance in future value terms rather than present value terms. The reason for this is that we want to capture the effect of risk-premia on our trading strategy. In this simplified model, there are two risk premia. The first is the premium of implied volatility over actuarial volatility. The second is the expected appreciation in the underlying equity market. In this case, since the growth rate is 15% and the risk-free rate is 6%, the embedded long equity risk premium is 9%. This risk premium would therefore affect the performance of hedging strategies involving futures. The figure below shows the equity risk premium embedded in futures positions. Essentially, the risk premium in futures is the same in dollar terms as the risk premium on the underlying index. Hence, any long futures position is expected to appreciate in value while any short position is expected to lose value.

**The expected return on futures is the risk premium on the stock.**

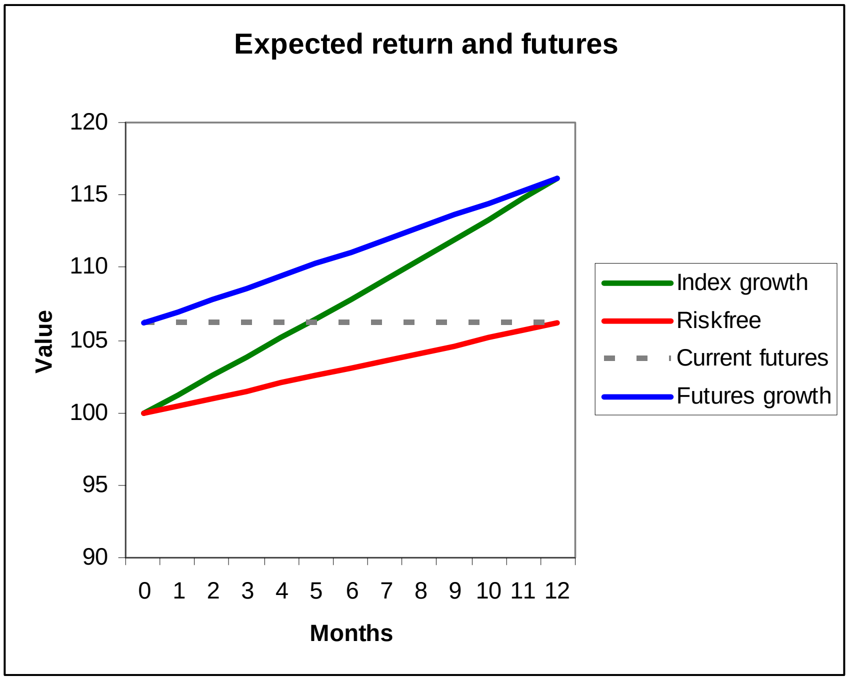

**Figure 1: Risk premia embedded in futures position**

The same logic applies to put options. Under the full reinsurance case, we are buying a put to cover our risk. On a delta basis, this is equivalent to shorting the market and going long volatility. Therefore, this strategy suffers the cost of both the equity market risk premium and the volatility risk premium.

Under the static hedge, we avoid the cost of the volatility risk premium, but pay the cost of the market risk premium because we are going short in order to hedge the short put position.

Under the dynamic hedge, we pay a market risk premium by going short the futures, but also avoid paying the volatility premium. Of course, we have assumed away any execution and operational risk.

Finally, if we do nothing, we do not pay market risk premia. If we stay short the put, on a delta basis, we are long the market and short volatility, effectively earning risk premia due to both factors.

Which strategy will outperform the others? How should the strategy be fine tuned? To answer these questions, we must address both risk and return considerations.

## RISK ANALYSIS

In order to compare the strategies side by side, we ran 2500 instances of the propagation simulation referenced earlier. We then ranked the outcomes according to the level of the underlying index at maturity. The resulting graph of the outcomes is shown below:

**Simulate terminal payouts of all strategies to obtain accurate comparisons.**

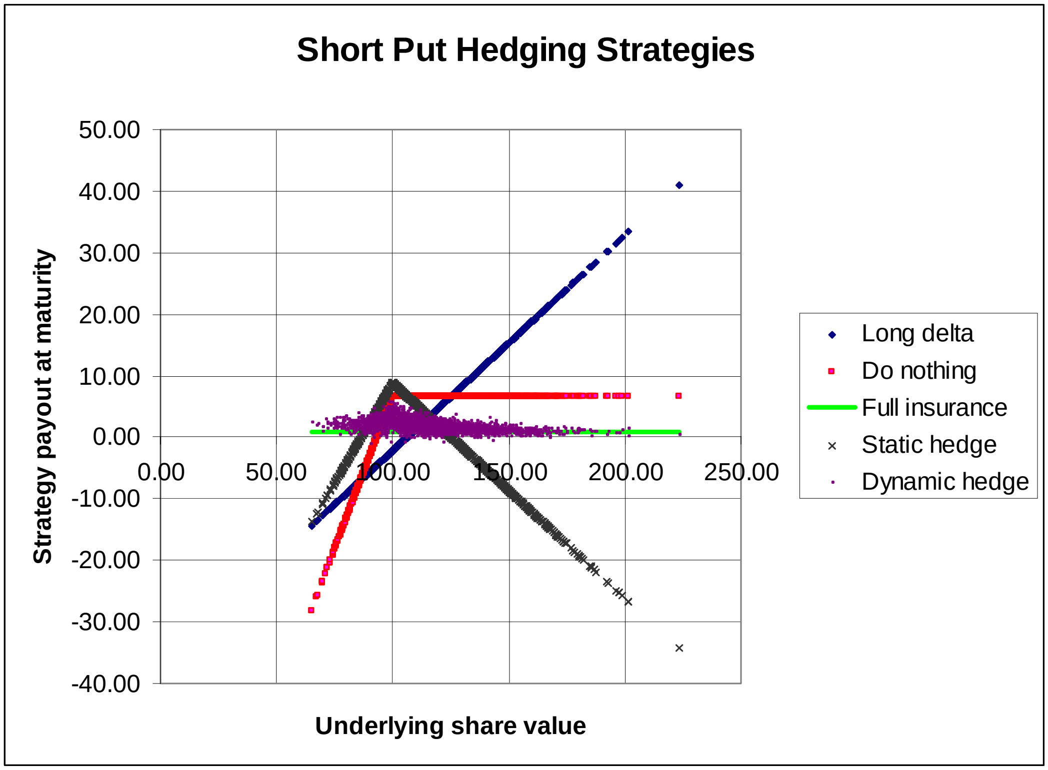

Figure 2: Simulated Terminal Payout Outcomes

The long delta strategy is shown for comparison purposes only; it is long 0.35 units of the underlying using the forward market. The "do nothing" option shows the classic profile in red of a short put option. By purchasing a market put option, we can lock in the payout at the green line. The static hedge in grey performs well in a down market, but leaves us vulnerable in a rapidly rising market. Finally, the dynamic hedging strategy results are not guaranteed, but they tend to hover in positive territory under the assumptions we've made in this example.

## PERFORMANCE MEASUREMENT

The benefit of this approach is that we can compare the expected value and risk of each of the strategies by subjecting them to a common measure: payout at maturity. In particular, this resolves the difficulty of comparing static and dynamic strategies in a theoretically elegant fashion.

To measure risk, we computed the following from the 2,500 simulations:

Risk measure = Expected profit – $5^{th}$ percentile profit

This is a downside risk measure that better accounts for asymmetric returns than the usual symmetric standard deviation measure.[3] When we plot expected profit versus risk, the following graph obtains:

**Hedging alternatives do not perform comparably.**

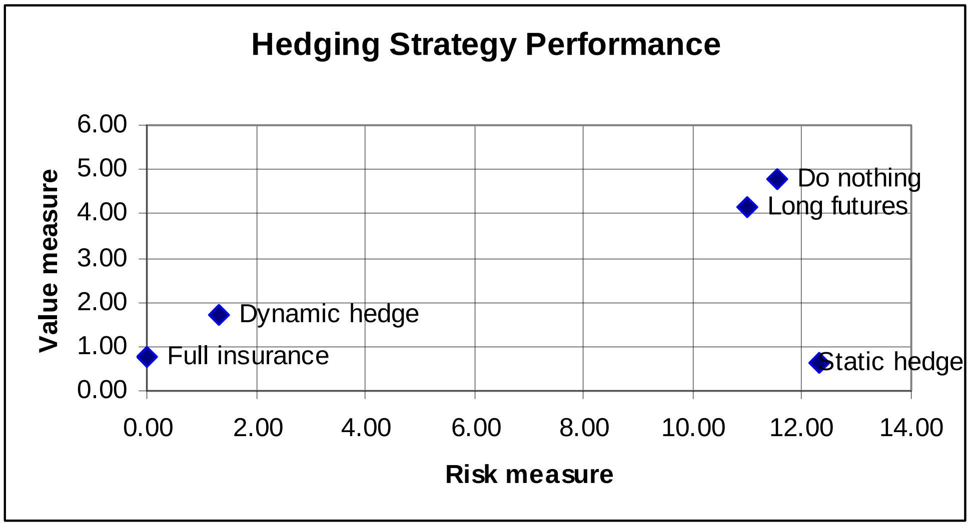

**Figure 3: Relative Performance Measures**

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[3] In other applications, this has been called CFAR, or "cash flow at risk".

Under these assumptions, the static hedge performs poorly, and is inferior to every other strategy.

How do we measure which strategy is best? One measure is the ratio of value to risk, which may be called the RAROC, or risk-adjusted return on capital. The RAROC in this graph is represented as the slope of the line from the origin to the point shown.

The highest RAROC is infinite in this example, suggesting full reinsurance is best. With an unlimited number of potential policies, this is clearly the best strategy. In our real world of limited policies, however, we have to ask the question if it is worthwhile to forego so much expected return. The dynamic hedge offers more profit per policy for a seemingly reasonable increment of risk. It may add more to shareholder value to follow a dynamic strategy, if the cost of the incremental risk is not too great.

**Do we want high expected returns, or high returns relative to risk?**

On the other hand, doing nothing offers the highest expected profit per contract, but at significant risk. It does seem to outperform going long the futures, however, which is possible because it earns the risk premium from being short volatility.

An alternative comparison measure for the various strategies can be determined by deducting a risk charge from the expected profitability. In practice, this would include statutory charges and margin liquidity costs as well. In this example, we may assume a 25% RAROC risk charge and determine the risk adjusted value-added (RAVA):

$$\text{RAVA} = \text{Expected profit} - \text{RAROC} \times \text{Risk measure}$$

| | Exp Profit | Risk | RAVA |
|---|---|---|---|
| Long futures | 4.14 | 11.00 | 1.39 |
| Do nothing | 4.78 | 11.55 | 1.89 |
| Full insurance | 0.79 | 0.00 | 0.79 |
| Static hedge | 0.64 | 12.32 | -2.44 |
| Dynamic hedge | 1.71 | 1.33 | 1.37 |

**Table 2: Terminal Payout Risk and Return Conclusion**

Under this measure, doing nothing adds the most value for shareholders, $1.89 per policy. Graphically, RAVA can be interpreted as the vertical distance from the performance point to the line whose slope is given by the cost of risk (RAROC). This can be seen in the following figure:

**RAVA reconciles return and risk measures to rank strategies.**

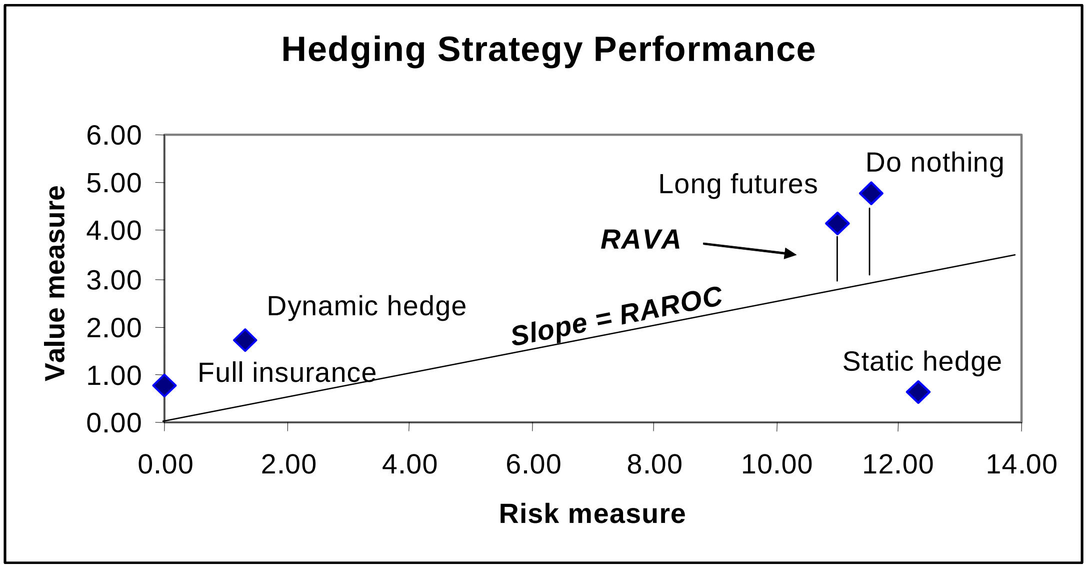

**Figure 3: RAVA and Relative Performance**

## CONCLUSION

This paper does not mean to suggest that VA writers should not manage their equity risk. Rather, it presents a simple methodology to compare the performance of complex static and dynamic strategies on the same basis. It can also be used to fine-tune the various hedging techniques. Finally, by allowing a realistic incorporation of well-established market risk premia, it gives the insurer an actuarial future view of risk and return results rather than the usual indifference determined by risk-neutral present value results.

The two methodologies are termed RAROC and RAVA, corresponding to risk-adjusted return on capital and risk-adjusted value-added, respectively. Either methodology can be rigorously supported and justified depending on the circumstances of the insurer.