

LIFE INSURANCE

SMART MODELING FOR A STOCHASTIC WORLD

Modern insurance company financial analysis requires use of stochastic methods. Tillinghast Smart Modelling™ makes them more manageable, usable and understandable.

By John Rowland and David Dullaway

Since the beginning of the decade, the global life insurance industry has made a lot of progress moving toward realistic valuation and reporting. In part, this is in response to a recognition of the value of financial options and guarantees embedded in many contracts. New methods based on concepts of modern financial theory, such as market-consistent embedded value and consistent risk, capital and value frameworks, are gaining ground. Stochastic modeling is proving especially helpful.

Initial attempts to adopt relatively simple closed form models to value instruments have been largely replaced with Monte Carlo simulation models linked to an economic scenario generator (e.g., Tillinghast's Economic Scenario Generator [ESG] application). While this development has contributed to significant improvements in models, it has resulted in a number of drawbacks, principally related to the slow and sometimes tortuous run times. The models are also complex to adjust and understand.

As a consequence, the depth of analysis and understanding that many companies previously took for granted is now burdensome. It has become difficult to update financials for today's market conditions or to run the numerous tests that would be required by varying the underlying assumptions to gain a real understanding of their

impact on results. The certainty that appeared to exist in the old deterministic world, which in practice ignored the stochastic nature of the underlying financial dynamics, has been replaced with stochastic uncertainty. Now new tools are required to make the analysis required less time-consuming and more understandable.

ANSWERS IN JUST 19,000 YEARS!

A good example of the mind-boggling numerical complexity arising in the stochastic world is provided by considering how a typical insurance company would simulate its balance sheet on a realistic basis. In a typical case, a Monte Carlo simulation model would be used; 40 years might be simulated, and 5,000 scenarios used to run the model. The numerical complexity arises if we assume that management decisions that are defined annually are based on realistic solvency at that time, requiring a value of the balance sheet at each year of the simulation. Therefore, to generate the value of the balance sheet at the beginning, we actually need to generate all future balance sheets, which equates to a staggering 40 x 5,000, or 200,000 balance sheets. The exhibit shows model run time under assumptions for the run time per scenario.

Given the timescales in the exhibit, it is easy to see why most companies have set up their models with management actions based on an approximation to the annual value of the balance sheet.

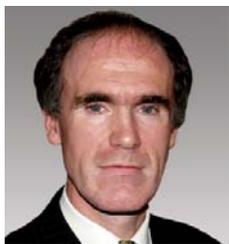
Technological developments will help to reduce these timescales. Grid computing could reduce run times significantly, albeit not sufficiently since it is clear that even a 1,000 grid is not going to be sufficient for this problem. Further, the historically true Moore's law, which states that computing power doubles every 18 months, implies much shorter run times (say on the order of one week in 30 years).

The key problem is a fundamental one: The typical insurance company is essentially using its traditional deterministic model to answer a modern stochastic question. A modern, smarter approach is required.

SMART MODELING

Tillinghast Smart Modelling™ provides the tool kit needed to live and operate in the stochastic world in which insurance companies find themselves. Developed, tested and proved with clients over the past three years, this approach enables clients to answer the traditional questions that the move to the stochastic world has rendered impractical, and also new questions that companies must address when they manage their risks in the new world.

At the heart of the approach is the recognition that significantly more information is available than is typically used from each set of Monte Carlo simulations.



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EXHIBIT

Run time for a realistic balance sheet with management actions based on annual realistic solvency

Per scenario	Per balance sheet	Per 200,000 balance sheets
1 second	83 minutes	32 years
10 seconds	14 hours	317 years
1 minute	3.5 days	1,900 years
10 minutes	34 days, 17 hours	19,000 years

Effective use of this additional information allows companies to benefit from:

- shorter run times without reducing the accuracy of the results
- more accurate results while running fewer scenarios
- better calibration of stochastic models to market data
- economic sensitivity results without the requirement to rerun the stochastic model.

Smart modeling is based on techniques that have been developed in the banking and finance industries. In many cases the key is developing a highly accurate approximation to a “replicating portfolio” for the liabilities — that is, a portfolio of assets whose behavior closely resembles that of the liabilities in question. Liability values, including values under economic stresses, can then be calculated by revaluing the replicating portfolio. This can be

carried out far more quickly and far more accurately than rerunning the existing stochastic models.

The smart modeling approach is implemented through tools that automate the process of finding approximate replicating portfolios, as well as tools that apply variance reduction techniques pioneered by the banking industry. There are also tools based on stochastic mesh techniques that enable values to be derived at future points of projections.

A number of key principles have guided the development of these tools:

- The tools rely on relatively standard theory that has been tried and tested in the financial literature and often throughout the banking industry. The applications have been thoroughly tested to ensure that they work for insurance problems as well as the banking problems they were developed for originally.

- Applications are model- and scenario-generator independent. They are designed to work with existing models and ESGs and typically would not require a model rebuild.

- Applications start from the output of a liability model run and use this together with the associated scenario file as inputs. Smart modeling is thus largely post-processing. This should reduce process error risks.

- The tools can be used on an ongoing basis without the need for significant ongoing external support.

SMART MODELING CASE STUDIES

Smart modeling is an evolution of stochastic Monte Carlo modeling, which enables insurance companies to meet the challenges they face in a stochastic world. It is highly likely that all stochastic modeling will incorporate some form of smart modeling in the future. The diversity of potential applications is reflected in the following case studies:

(1) Model speed transformation: The most common application of Tillinghast Smart Modelling to date has been speeding up the production of embedded values and balance sheets generated by insurer valuation models in Germany, Japan and the U.K. A typical example uses the replicating portfolio tool to construct a replicating portfolio for the company’s liabilities and

Smart modeling is implemented through tools that automate selection of replicating portfolios and that apply variance reduction techniques.

then using this as a control variate in the “variance reduction tool” to enable companies to run their stochastic models with a reduced number of scenarios. Typically, up to 20 times fewer scenarios are required to achieve the same level of accuracy. This means that companies that currently run a year-end reporting process over a period of weeks can complete all model runs in a single day and spend the remaining time investigating and interpreting the results. The analysis has been audited and subject to regulatory scrutiny. Tillinghast Smart Modelling tools have been licensed by companies for their own use at future reporting dates.

(2) Developing hedging tools: A second important application of smart modeling is its relevance to managing and monitoring hedging programs. As companies have developed a better understanding of the economic risks embedded in their balance sheets, many have opted for hedging programs with daily or weekly monitoring of the hedge. The monitoring requires the derivation of “Greeks” that involve multiple runs of stochastic models. This is feasible given the faster model run-time speeds available from smart modeling using the approaches set out above. However, a better solution goes back to the idea of using closed form approaches that can be valued without resorting to Monte Carlo simulation. The ideal closed form model is a replicating portfolio of the liabilities that

Mathematically speaking...

How does smart modeling achieve variance reduction?

Monte Carlo simulation is the process of deriving a valuation using an approximation (a finite simulation) to an expected value. Suppose that Y_i is the i^{th} simulation of the value (the expectation of Y or $E(Y)$) to be derived. In our applications, Y_i is the discounted value of the liability cash flows under the conditions of scenario i . If the observations Y_i are independent and identically distributed (i.i.d.), then the sample mean Y^n (i.e., the average of (Y_1, Y_2, \dots, Y_n)) is unbiased and converges to the desired value ($E(Y)$) with a probability of 1. Suppose that we calculate another variable, X_i , along with Y_i , so that the pair is i.i.d. for $i = 1, 2, \dots, n$ and the expectation $E(X)$ is known. The variable X corresponds to the replicating portfolio of the liabilities. Thus X_i is the value of the replicating portfolio along the i^{th} scenario and $E(X)$ is known, since the replicating portfolio can be valued in closed form.

For any b , calculate: $Y_i(b) = Y_i - b(X_i - E(X))$

Then for any b , the sample mean of the variable $Y_i(b)$ (defined as the average of $Y_1(b), Y_2(b), \dots, Y_n(b)$) is an unbiased estimator of the target value ($E(Y)$).

The variable $Y_i(b)$ can be used for variance reduction. Since $E(X)$ is known, when simulating X using Monte Carlo simulation, the simulation error of X along each scenario, defined by $(X_i - E(X))$, is known. If X and Y are correlated, then it is reasonable to assume that the simulation errors of X and Y along each scenario will be correlated. Therefore, we should be able to use the simulation error in X (namely $(X_i - E(X))$) as an error correction for the simulation error in Y .

An optimal adjustment is made by choosing the parameter b to minimize the variance of the sample mean of $Y_i(b)$. It can be shown* that the choice $b = \text{Cov}(X, Y) / \text{Var}(X)$ is optimal and that the variance of the controlled simulation is reduced by a factor of $1 / (1 - \rho^2)$ where ρ is the correlation between X and Y .

This method of variance reduction when using Monte Carlo simulation is referred to as using (linear) control variates (or variables). The key for optimal variance reduction when undertaking Monte Carlo simulation is the existence of controls X that are highly correlated with the target variable Y .

*Glasserman P., *Monte Carlo Methods in Financial Engineering*, Springer, USA, pages 185-187.

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can be valued in closed form, enabling Greeks to be calculated directly. The replicating portfolio tool is designed to construct such a portfolio.

(3) Implementing a real-time risk dashboard: The realistic reporting regime in the U.K. and the advent of Solvency II are placing ever greater emphasis on a company's ability to link financial and risk management. This is creating demand for real-time risk reporting. One U.K. insurer wanted monthly reporting with a risk dashboard for its with-profit funds, which was implemented using a smart modeling approach. The resulting dashboard reports key financial ratios based on realistic balance sheet and capital positions. The report is available on day two of the month without the need to run stochastic models. Such a report is relatively easy to implement if a replicating portfolio of the liabilities can be found using the smart modeling replicating portfolio tool.

(4) Capital projections — reducing run time from 19,000 years to a few hours: We illustrated earlier how the derivation of balance sheet values at each year of a stochastic simulation could result in astronomical run times. There is a practical solution to an equivalent problem: the projection of the realistic capital runoff for with-profit liabilities.

In this instance, the solution is based on an application of stochastic mesh techniques that enable future values to be estimated within a projection.

WHERE NEXT?

Tillinghast Smart Modelling provides the required tool kit that makes it practical for companies to meet their financial reporting and information requirements, even in a stochastic world. But this is just the beginning. We expect to see a time when all stochastic modeling will be undertaken using techniques that are essentially based on smart modeling approaches, with many additional applications soon to come.

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NEWS

Tillinghast Again Named "Best Consultancy of the Year"

The Tillinghast insurance consulting practice of Towers Perrin was once again named "Best Consultancy of the Year" by *Reactions* magazine, winning the award for insurance professionals in this category for the third time.

"This is a testimony to our people — the consultants who consistently live Towers Perrin values as a global professional services firm focused on providing our clients the highest-quality work to strengthen their businesses," said Americas P/C Insurance Practice Leader, Jeanne Hollister. "We truly appreciate this acknowledgment that reflects our reputation and our relationships with our clients."

Towers Perrin Appoints New Software Sales Director

Towers Perrin announced that David Tonner has joined the firm's Tillinghast business to direct sales of financial modeling software across EMEA. He has over 20 years' experience in the area of software sales and systems integration projects, with a specialization in the life insurance and retail banking industries.

In his new London-based role, David will focus on working with Towers Perrin financial modeling products and specifically on the newly developed risk management software framework, RiskAgility™.